Square, we get

$$(c-r)^2 = R^2 \qquad [\text{as } n\cdot n=1]$$

Let $F(s) = (c-r)^2 - R^2$

For four point contact, we have

$$F'(s)=0,\quad F''(s)=0,\quad F'''(s)=0,\quad F^{iv}(s)\neq 0$$

then $F'(s) = 2(c-r)\cdot(-t) = 0$

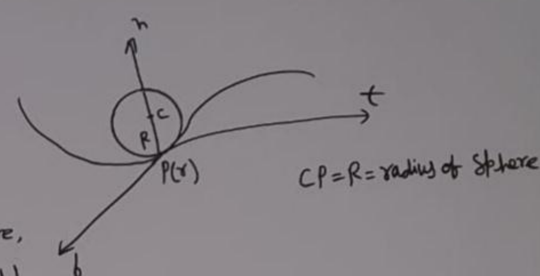
but $-2\neq 0$

$$\therefore (c-r)\cdot t = 0 \quad \text{— (1)}$$

Again $F''(s) = (c-r)\cdot t' - t\cdot t = 0$

Osculating Sphere:

The osculating sphere is the sphere which has four point contact with the curve at P.



Let C is the centre of osculating sphere, and R is the radius of sphere and (t, n, b) are unit vector at P. Then

$$\vec{PC} = (c-r)$$

and also $\vec{PC} = Rn$

$$\Rightarrow Rn = (c-r)$$

$$(\alpha n + \mu b)\cdot n = \rho$$

$$\Rightarrow \alpha = \rho$$

and $$(\alpha n + \mu b)\cdot(\tau b - \kappa t) = \rho'$$

$$\Rightarrow \mu\tau = \rho'$$

or $$\mu = \frac{\rho'}{\tau} = \rho'\sigma$$

Hence put in (4)

$$c - r = \rho n + \rho'\sigma \cdot b$$

or $$\boxed{c = r + \rho n + \rho'\sigma b}$$

The radius of spherical curvature is given by

$$\boxed{R = \rho^2 + \sigma^2\rho'^2}$$ since $c - r = R$

$$\Rightarrow (c - r)\kappa n - 1 = 0$$

$$\Rightarrow (c - r)\cdot n = \frac{1}{\kappa} = \rho \quad \text{— (2)}$$

diff (2)

and $$F'''(s) = (c - r)n' - n\cdot t = \rho'$$

$$\Rightarrow (c - r)(\tau b - \kappa t) - 0 = \rho' \qquad [\text{as } n\cdot t = 0$$

$$\Rightarrow (c - r)\cdot(\tau b - \kappa t) = \rho' \quad \text{— (3)}$$

Eqn (1) Show that $c - r$ lies in the normal plane, show that

$$c - r = \alpha n + \mu b \quad \text{— (4)}$$

where α and μ are scalars.

Substituting this value of $(c - r)$ in (2) and (3),

we get